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MAY 80 R J DUFFIN DAAG29-77-G-0024

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THE AEMS PROJECT IN APPLIED MATHEMATICS AT CARNEGIE-MELLON UNIVERSITY

Comprehensive Report
Period 1970 - 1980

prepared for

Research Grants: DAAG29 77 G 0024
DA-ARO-D-31-142-71-G17; 73-G131
Army Research Office
Research Triangle Park,
North Carolina

by

Dr. R. J. Duffin
Carnegie-Mellon University
Pittsburgh, Pennsylvania

May, 1980

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The AEMS Project in Applied Mathematics at Carnegie-Mellon University

I. Descriptions and goals.

This is a record of a research project in applied mathematics at Carnegie-Mellon University. The project is called, "Analysis of Electrical and Mechanical Systems" or AEMS for short. Since 1951 the project has received continued support from the Army Research Office, Research Triangle Park, North Carolina.

This report is an account of the project for the period 1970-1980. During that time the project has been awarded Grants: DAAG 29 77 G 0024; DA-ARO-D-31-142-71-G17; 73-G131. This is a continuation of a previous report entitled Comprehensive Final Report, Period 1951-1970.

The principal investigator of the AEMS Project is R. J. Duffin, University Professor of Mathematical Sciences at C-MU. Professor Duffin received the B.S. degree in engineering and the Ph.D. degree in physics both from the University of Illinois. He serves as associate editor for several journals of applied mathematics. He is a member of the National Academy of Sciences and of the American Academy of Arts and Sciences. He is a part-time consultant to the Westinghouse Research Laboratories. His teaching duties at C-MU involve undergraduate lecture courses in applied mathematics and supervision of M.S. and Ph.D. students.

The aims of ~~AEMS~~ the Analysis of Electrical and Mechanical Systems Project are threefold:

~~(a)~~ to apply mathematical analysis to new problems arising in science and technology. → next page

(b) to develop new mathematical concepts and structures using facets of science and technology as models; and

(c) to help students start a career in the interesting area of applied mathematics. Translated [illegible] published since 1970. Abstracted are [illegible]

II. <u>Scientific</u> <u>personnel</u>.

A good share of the scientific contribution of the project resulted from the work of graduate students who served as research assistants. The students who received the Ph.D. degree since 1970 are:

S. Bhargava - Mysore University, India

Jeff Buckwalter - Bank of America, San Francisco

Victor Burke - University of Toledo

Patrick Hayes - Federal Reserve, Washington, D.C.

Thomas Morley - University of Illinois, Urbana

George Trapp - West Virginia University

Maurice Weir - Naval Postgraduate School, Monterey

Masters degree students include

Pamela Brickman - Chase National Bank, New York City

Steve Hooper - Mellon Bank, Pittsburgh

Robert Smith - Hewlett-Packard Company

In addition George Polak and Robert Merkovsky are now preparing for the Ph.D.

The research on the AEMS project involved a great deal of collaboration with scientists at C-MU and at other institutions as well. Moreover, some of these scientists came to C-MU to spend a sabbatical year. These were:

Professor T. Shimpuku
Department of Physics
Shiga College
Shiga, Japan

Professor S. Tu
Department of Mathematics
National Central University
Chung Li, Taiwan

Professor T. Nishizeki
Faculty of Engineering
Tohoku University
Sendai, Japan

A list of people collaborating on the scientific work of the project follows:

W. N. Anderson
M. F. Barnsley
S. Bhargava
C. E. Blair
P. J. Brehm
J. Buckwalter
V. Burke
C. V. Coffman
B. D. Coleman
P. R. Gribik
T. R. Jefferson
R. G. Jeroslow
L. A. Karlovitz
G. P. Knowles
K. O. Kortanek
D. N. Lee
W. H. McWhirter
V. J. Mizel
T. D. Morley
T. Nishizeki
J. J. Oravec
E. L. Peterson
G. C. Polak
J. S. Schruben
D. H. Shaffer
T. Shimpuku
R. Smith
G. E. Trapp
S. Tu
C. Zener

III. Interaction with the scientific community.

The research results have been reported at a number of national and international meetings. These include invited talks at meetings of the following societies: AMS, SIAM, ORSA, TIMS, and IEEE. Some examples are listed here:

"Vibrations of a beaded string analyzed topologically", Conference on Mapping Techniques, Houston, Texas, November 1970.

"Duality inequalities of mathematics and science", Symposium on Nonlinear Programming, Mathematics Research Center, Madison, Wisconsin, April 1971.

"Extension of Geometric Programming" presented at the International School on the Impact of Optimization Theory on Technological Design, Leuwen Belgium, August 1971.

"Some problems of mathematics and science", an hour talk at the annual meeting of the AMS, San Francisco, January 1974.

"Extremal problems of applied mathematics", The Arthur Coble Lecture, Department of Mathematics, University of Illinois, Urbana Illinois. September 1974.

"Are adobe walls optimal phase shift filters", Tenth International Symposium on Mathematical Programming, Montreal, Canada, August 1979.

A lively conference, Constructive Approaches to Mathematical Models, was held at Carnegie-Mellon University, July 10-14, 1978. This conference was in honor of R. J. Duffin. There were over fifty papers presented by mathemeticians and engineers. The papers were in the area of Graphs and Networks, Mathematical Programming, Differential Equations and Mathematical Models. The proceedings of the conference was published by Academic Press in 1979.

IV. Publications of the AEMS project.

There follows a list of papers published since 1970. The topics studied were quite varied so rather than attempting to summarize the total work, an abstract of each paper is given. There is, however, an underlying theme in that most of the papers involve a mathematical model of a physical or economic situation.

1. "Network models"
Mathematical Aspects of Electrical Network Theory, SIAM-AMS Proceedings 3(1971), 65-91.

The steady flow of electrical current through a network of conductors has served as a suggestive model for a variety of mathematical theories. This paper describes electrical models related to the following theories; series-parallel graphs, parallel addition of matrices, lattice theory, generalized inverses, Grassmann algebra. Wang algebra, matroids, extremal length. Rayleigh's reciprocal relation and the width-length inequality.

2. "Geometric programming and the Darwin-Fowler method in statistical mechanics"
Journal of Physical Chemistry 74(1970), 2419-2423 (with C. Zener).

This paper concerns the classical problem of chemical equilibrium as formulated in the language of geometric programming. Thus the equilibrium state at constant temperature and volume is characterized by the duality principle, minimum F =

maximum F^*. Here F is the Helmholtz function for free energy and F^* is a new function termed the anti-Helmholtz function. The minimization of F is constrained by the mass balance equations. However the maximization of F^* is unconstrained. Hence this gives a simplified practical procedure for calculating equilibrium concentration. The chemical equilibrium can also be analyzed by statistical mechanics. Comparing the two methods brings to light an intimate relationship between geometric programming and Darwin-Fowler statistics.

3. "Equipartition of energy in wave motion"
Journal of Mathematical Analysis and Applications 32(1970), 386-391.

Of concern are solutions of the classical wave equation in three-dimensions. It is shown that if a solution has compact support then after a finite time, the kinetic energy of the wave is constant and equals the potential energy. The proof employs the Paley-Wiener theorem of Fourier analysis.

4. "Duality inequalities of mathematics and science"
Nonlinear Programming, Edited by J. B. Rosen, O. L. Mangasarian, and K. Ritter, Academic Press, New York, 1970, 401-423.

The problem of minimizing a scalar functional $u(x)$ under a set of constraints S in the vector variable x is termed a program. It often results that there is an associated program of maximizing a scalar functional $v(y)$ under a set of constraints T on the vector y. These programs are termed dual if it can be shown that the functional $u(x)$ exceeds the functional $v(y)$ Then there exists a constant M such that

$$\underset{x \in S}{u(x)} \geq M \geq \underset{y \in T}{v(y)}.$$

The virtue of this duality inequality is that it permits estimating M with a known bound on the error. Inequalities of this form appear in various areas of mathematics, science, engineering, and economics. This paper points out several such duality inequalities and their interrelationships.

5. "Yukawan potential theory"
Journal of Mathematical Analysis and Applications 35(1971), 70-13.

This paper concerns the Yukawa equation $\Delta u = \mu^2 u$ where μ is a real constant. Given a solution $u(x,y)$ of this equation then there is a conjugate function $v(x,y)$

satisfying the same equation and related to $u(x,y)$ by a generalization of the Cauchy-Riemann equations. This gives rise to interesting analogies with logarithmic potential theory and with complex function theory. In particular there are generalizations of holomorphic functions, Taylor series, Cauchy's formula, and Rouche's theorem. The resulting formulae contain Bessel functions instead of the logarithmic functions which appear in the classical theory. However, as $\mu \to 0$ the formulae revert to the classical case. A convolution product for generalized holomorphic functions is shown to produce another generalized holomorphic function.

6. "Vibration of a beaded string analyzed topologically"
Applicable Analysis 56(1974), 287-293.

Of concern are the transverse vibrations of a finite string of beads. It is shown that a periodic vibration can result when the beads are released from an initial configuration. Moreover a norm on the initial configuration can be given a prescribed value. The proof uses the Brouwer fixed point theorem.

7. "Geometric programming with signomials"
Journal of Optimization Theory 11(1973), 3-35 (with E. L. Peterson).

The difference of two "posynomials" (namely, polynomials with arbitrary real exponents, but positive coefficients and positive independent variables) is termed a "signomial".

Each signomial program (in which a signomial is to be either minimized or maximized subject to signomial constraints) is transformed into an equivalent posynomial program in which a posynomial is to be minimized subject only to inequality posynomial constraints. The resulting class of posynomial programs is significantly larger than the class of (prototype) posynomial programs in which a posynomial is to be minimized subject only to upper-bound inequality posynomial constraints. However, much of the (prototype) geometric programming theory is generalized by studying the "equilibrium solutions" to the "reversed geometric programs" in this larger class.

8. "Hybrid addition of matrices - A network theory concept"
Applicable Analysis 2(1972), 241-254 (with G. E. Trapp).

The parallel connection of networks suggested the concept of parallel addition of matrices to Anderson and Duffin. The hybrid connection of networks also suggests a matrix operation. Using the Kirchhoff current and voltage equations, a new operation, hybrid addition, is defined for the set of Hermitian semidefinite matrices. This operation is an Hermitian semidefinite order preserving semigroup operation. Hybrid addition is closely related to the work of Anderson on "shorted operators", and to the gyration operation of linear programming and network synthesis.

9. "Geometric programs treated with slack variables" Applicable Analysis 2(1972), 255-267 (with E. L. Peterson).

Kochenberger and Woolsey have introduced slack variables into the constraints of a geometric program and have added their reciprocals to the objective function. They find this augmented program advantageous for numerical minimization. In this paper the augmented program is used to give a relatively simple proof of the "refined duality theory" of geometric programming. This proof also shows that the optimal solutions for the augmented program converge to the (desired) optimal solutions for the original program.

10. "Reversed geometric programs treated by harmonic means" Indiana University Mathematics Journal 22(1972), 531-550 (with E. L. Peterson).

A "posynomial" is a (generalized) polynomial with arbitrary real exponents, but positive coefficients and positive independent variables. Each posynomial program in which a posynomial is to be minimized subject to only inequality posynomial constraints is termed a "reversed geometric program".

The study of each reversed geometric program is reduced to the study of a corresponding family of approximating (prototype) "geometric programs" (namely, posynomial programs in which a posynomial is to be minimized subject to only upper-bound inequality posynomial constraints). This reduction comes from using the classical arithmetic-harmonic mean inequality to "invert" each lower-bound inequality constraint into an equivalent "robust" family of "conservatively approximating" upper-bound inequality constraints. The resulting families of approximating geometric programs are then studied with the aid of the techniques of (prototype) geometric programming.

11. "Adjacency matrix concepts for the analysis of the interconnection of networks" Journal Franklin Inst. 298(1974), 9-27 (with W. N. Anderson, Jr. and G. E. Trapp).

A mathematical theory is developed for an imagined device termed a "junctor". A junctor could be used to interconnect two n-terminal networks giving rise to another n-terminal network. Actually a junctor is itself a simple network with three banks of n terminals internally connected in some fashion by perfectly conducting wires. Incidence matrices are formulated to analyze various junctors and their current flows. The main problem treated concerns conditions which ensure that the junctor operation is associative.

12. "Network models for maximization of heat transfer under weight constraints"
Journal of Networks 2(1972), 71-48 (with S. Bhargava).

Of concern is a network in which the conductance of certain branches are variable. The problem posed is the maximization of the joint conductance subject to a bound on the ℓ_ρ norm of the variable conductances. It is shown that at an optimum state the conductance of a variable branch is proportional to the $2/(\rho + 1)$ power of the current through the branch. This relation together with a dual variational principle leads to a "duality inequality" giving sharp upper and lower estimates of the maximum joint conductance. Such a network serves as a discrete model for a cooling fin subject to a weight limitation. Thus the model shows what analogous properties should hold for the cooling fin.

13 "Dual extremum principles relating to cooling fins"
Quarterly of Applied Mathematics 31(1973), 27-41 (with S. Bhargava).

Under consideration is a differential equation $(pu')' = qu$ of the Sturm-Liouville type where the function $q(x) > 0$ is given. The problem is to find a function $p(x) > 0$ in $0 < x < b$, a constant b and a solution $u(x)$ of the corresponding differential equation such that the energy functional

$$\int_0^b [p(u')^2 + qu^2]\,dx$$

is maximized when $p(x)$ is subject to the constraint $\int_0^b p^\rho dx \leq K^\rho$ and u is subject to the boundary conditions $u = 1$ at $x = 0$ and $p\frac{du}{dx} = 0$ at $x = b$. Here $K > 0$ and $\rho \geq 1$ are constants. A pair of dual extremum principles is found to give sharp upper and lower estimates of the maximum value of the energy functional.

14. "Dual extremum principles relating to optimum beam design" Archive for Rat. Mech. and Analysis 50(1973), 314-330 (with S. Bhargava).

Of concern is a cantilever beam resting on an elastic foundation and supporting a load at the free end. The beam is of rectangular cross section and of constant height but variable width. It is required to taper the beam for maximum strength. This means that the beam is to support a maximum vertical load W at the free end when the free end is given unit deflection. The constraint is that the weight of the beam should not exceed a given bound K. It is shown that the optimum taper should be so chosen that the curvature of the beam is constant. This yields the solution of the problem in terms of explicit formulas. For more general constraints, an inequality is found which gives upper and lower bounds for the maximum load W even though explicit formulas are not available.

15. "On the nonlinear method of Wilkins for cooling fin optimization" SIAM J. Appl. Math. 24(1973), 441-448 (with S. Bhargava).

Of concern is the nonlinear differential equation equation $(k(u)p(x)u')' = p^{\eta}(x)Q(u) \quad 0 \leq x \leq b$ subject to the boundary conditions: $u = 1$ at $x = b$ and (kpu') takes the values $y_0 > 0$ and 0 at $x = b$ and $x = 0$ respectively. Here $0 \leq \eta < 1$ is a given constant and k and Q are known functions and the question posed is to find a positive constant $b > 0$, a function $p(x) > 0$ in $0 < x \leq b$ and a solution u of the differential equation such that the norm $\left(\int_0^b p^{\rho}dx\right)^{1/\rho}$, $p \geq 1$ is minimized. A special transformation of variables together with Hölder's inequality leads to the solution in terms of explicit quadrature formulas.

16. "Matrix operations induced by network connections" SIAM J. Control 13(1975), 446-461 (with W. N. Anderson and G. E. Trapp).

In this paper a confluence is defined as a subspace of 3n-dimensional space having an indefinite inner product with signature (n,n,-n). Physically a confluence represents the vector space of all currents allowed by a given network interconnection. The space of voltages is then the orthogonal complementary confluence.

17. "Convex analysis treated by linear programming"
Journal of Mathematical Programming 4(1973), 125-143.

The theme of this paper is the application of linear analysis to simplify and extend convex analysis. The central problem treated is the standard concex program - minimize a convex function subject to inequality constraints on other convex functions. The present approach uses the support planes of the constraint region to transform the convex program into an equivalent linear program. Then the duality theory of infinite linear programming shows how to construct a new dual program of bilinear type. When this dual program is transformed back into the convex function formulation it concerns the minimax of an unconstrained Lagrange function. This result is somewhat similar to the Kuhn-Tucker theorem. However, no constraint qualifications are needed and yet perfect duality maintains between the primal and dual programs.

18. "The proximity of (algebraic) geometric programming to linear programming"
Mathematical Programming 3(1972), 250-253, (with E. L. Peterson).

Geometric programming with (posy)monomials is known to be synonomous with linear programming. This note reduces algebraic programming to geometric programming with (posy)binomials.

19. "Tripartite graphs to analyze the interconnection of networks"
<u>Graph Theory and Applications</u>, Edited by Y. Alavi, D. R. Lick, and A. T. White, 1972, 7-12.

In electrical network theory, many properties of connected networks are determined primarily by the connection and not the particular components that are connected. In this paper, we begin by viewing the interconnection of networks as a graph defined on three sets of vertices. By considering the networks as graphs, we are able to employ the concepts of adjacency matrices. We obtain results concerning interconnected graphs that are independent of our electrical network model.

20. "Nonuniformly elliptic equations: positivity of weak solutions"
Bulletin of the American Mathematical Society 79(1973), 496-499 (with C. V. Coffman and V. J. Mizel).

This is a study of a class of degenerate second order elliptic operators. It is shown that: (i) the first eigenfunction and the Green's function are strictly positive and (ii) a non-negative eigenfunction necessarily minimizes the Rayleigh quotient on the appropriate function space. The motivation for this study was a certain nonlinear eigenvalue problem.

21. "Positivity of weak solutions of non-uniformly elliptic equations" Annali di Matematica pura ed applicata CIV(1975), 209-238, (with C. V. Coffman and V. J. Mizel).

Let A be a symmetric $N \times N$ real-matrix-valued function on a connected region Ω in R^n with A positive definite a.e. and A, A^{-1} locally integrable. Let b and c be locally integrable, non-negative, real-valued functions on Ω, with with c positive, a.e. Put $a(u,v) = \int_\Omega ((A\nabla u, \nabla v) + buv)dx$. We consider the boundary value problem $a(u,v) = \int_\Omega fvcdx$, for all $v \in C_0^\infty(\Omega)$. and the eigenvalue problem $a(u,v) = \lambda \int_\Omega uvcdx$, for all $v \in C_0^\infty(\Omega)$. Positivity of the solution operator for the boundary value problem, as well as positivity of the dominant eigenfunction (if there is one) and simplicity of the corresponding eigenvalue are proved to hold in this context.

22. "Parallel subtraction of matrices" Proc. Nat. Acad. Sci. USA 69(1972), 2530-2531, (with W. N. Anderson and G. E. Trapp).

A new Hermitian semidefinite matrix operation is studied. This operation - called parallel subtraction - is developed from the theory of paralled addition. Since the theory of parallel addition is motivated by the analysis of interconnected electrical networks, parallel subtraction may be interpreted in terms of the synthesis of electrical networks. The idea of subtraction is also extended to hybrid addition.

23. "Hilbert transforms in Yukawan potential theory"
Proc. Nat Acad. Sci USA 69(1972) 3677-3679.

If H denotes the classical Hilbert transform and $Hu(x) = v(x)$, then the functions $u(x)$ and $v(x)$ are the values on the real axis of a pair of conjugate functions, harmonic in the upper half-plane. This note gives a generalization of the above concepts in which the Laplace equation $\Delta u = 0$ is replaced by the Yukawa equation $\Delta u = \mu^2 u$ and in which the Cauchy-Riemann equations have a corresponding generalization. This leads to a generalized Hilbert transform H_μ. The kernel function of this new transform is expressable in terms of the Bessel function K_0. The transform is of convolution type.

24. "An integral equation formulation of Maxwell's equations"
Journal of Franklin Inst. 289(1974) 385-394 (with J. H. McWhirter)

A classical method for solving static field problems is based on Fredholm integral equations. Here we consider the applications of integral equations to the general electromagnetic problem when the applied fields are alternating. Attention is focused on a problem with cylindrical symmetry. By employing Green's third identity, the boundary value problem is turned into a pair of integral equations of the second kind. This set of equations can form the basis for the numerical solution of these problems.

25. "A differential equation describing biological cell growth inhibition"
(with J. Schubert).

A phenomenological model for the inhibition of cell growth by a chemical agent is formulated. This leads to a formula relating the cell doubling time to the concentration of the agent. This formula gives good agreement with experimental data. An extension of this formula is proposed to treat a mixture of agents.

26. "On Fourier's analysis of linear inequality systems"
Math. Programming Study 1(1974) 71-94.

Fourier treated a system of linear inequalities by a method of elimination of variables. This method can be used to derive the duality theory of linear programming. Perhaps this furnishes the quickest proof both for finite and infinite linear programs. For numerical evaluation of a linear program, Fourier's procedure is very cumbersome because a variable is eliminated by adding each pair of inequalities having coefficients of opposite sign. This introduces many redundant inequalities. However, modifications are possible which reduce the number of redundant inequalities generated. With these modifications the method of Fourier becomes a practical computational algorithm for a class of parametric linear programs.

27. "Some problems of mathematics and science"
Bulletin of the American Mathematical Society 80(1974), 1053-1070.

The development of mathematics has often been aided by the use of models from science and technology. There are three main reasons why models help: (i) attention is focused on significant problems; (ii) the intuition is aided in perceiving complex relations; (iii) new concepts are suggested. This paper describes problems arising from models which have interested me. The models come from physics, chemistry, engineering, and economics.

28. "Nonlinear networks IIa"
Nonlinear Networks: Theory and Analysis, Edited by A. N. Wilson, IEEE Press (1975), 29-37.

This is a reprint of a paper on the steady flow of current in an electrical network. It is shown that if the resistors have a monotone characteristic then a unique flow exists.

29. "Lagrange multiplier method for convex programs"
Proc. Nat. Acad. Sci. USA 72(1975), pp. 1778-1781.

The problem of minimizing a convex function that is subject to the constraint that a number of other convex functions be non-positive can be treated by the Lagrange multiplier method. Such a treatment was revived by Kuhn and Tucker and further studied by many other scientists. These studies led to an associated maximizing problem on the Lagrange function. The aim of this note is to give a short elementary proof that the infimum of the first problem is equal to the supremum of the second problem. To carry this out it is necessary to relax the constraints of the first (or the second) problem so that the constraints are enforced only in the limit. This relaxation of constraints is not necessary should prescribing upper bounds to all the points in the domain of the functions. The domain of the functions can be n-dimensional space or a reflexive Banach space.

30. "Electrical network models"
Studies in Graph Theory, Edited by D. R. Fulkerson, MAA Studies 11(1975), pp. 94-138.

The history of science shows that the development of mathematics has been accelerated by the use of models. Thus geometric diagrams have served as models for algebraic relations. Gambling has served as a model for probability theory. Gravitation has served as a model for harmonic functions. Such models have accelerated mathematical development for three main reasons: (i) Attention is focused on significant problems. (ii) Models aid the intuition in perceiving complex relations. (iii) New concepts are suggested.

31. "Distortionless wave propagation in inhomogeneous media and transmission lines"
Quarterly of Appl. Math. 34(1976) pp. 183-194 (with V. Burke and D. Hazony).

Of concern are mechanical or electrical waves in a media which may be nonuniform and dissipative. The problem posed is to find conditions for the undistorted propagation of signals. The electrical transmission line is chosen as the general model. Along the length of the transmission line there are four functions which may be prescribed essentially arbitrarily. These are series resistance, series inductance, shunt conductance, and shunt capacitance. A differential equation is derived relating these functions which gives a necessary and sufficient requisite for distortionless transmission of a voltage wave. Various corollaries of this theorem are developed. For instance, it is shown that simultaneous voltage and current waves can be transmitted without distortion if and only if the characteristic impedance of the transmission line is positive at each point.

32. "Orbits of most action on a convex billiard table"
Houston J. of Math. 2(1976) pp. 453-470 (with L. A. Karlovitz).

Of concern are the trajectories of a ball bouncing inside a convex enclosure. The ball is treated as a mass point but the Coriolis force is not neglected. Thus between bounces a trajectory is a circular arc of given radius. At a bounce the angles of incidence and reflection are equal. Complete trajectories are analyzed from two different points of view. The first relies on a static model of the dynamic situation. The second is based on a version of the Jacobi least action principle and the notion of total action. By either analysis it is shown for example, that there are convex periodic orbits having a prescribed number of bounces. Another problem treated is the determination of a

Coriolis ellipse (a convex enclosure having two focal points).

33. "Convex programs having some linear constraints"
Proc. Natl. Acad. Sci. USA 74(1977) pp. 26-28.

The problem of concern is the minimization of a convex function over a normed space (such as a Hilbert space) subject to the constraints that a number of other convex functions are not positive. As is well known, there is a dual maximization problem involving Lagrange multipliers. Some of the constraint functions are linear, and so the Uzawa, Stoer, and Witzgall form of the Slater constraint qualifications is appropriate. A short elementary proof is given that the infimum of the first problem is equal to the supremum of the second problem.

34. "Algorithms for localizing roots of a polynomial and the Pisot Vijayaraghavan numbers"
Pacific J. of Math. 74(1978) pp. 47-56.

Pisot and Vijayaraghavan studied numbers whose mth power is nearly an integer in the sense that the discrepancy vanishes as m becomes infinite. One plus square root two is an example. Algebraic numbers of this type are characterized as algebraic integers whose conjugate roots each have absolute value less than one. This note develops a test for this property. An algorithm is given which determines whether or not one root of a polynomial has absolute value greater than one and all the other roots have absolute value less than one. If n is the degree of the polynomial, this algorithm involves only n rational steps.

35. "Almost definite operators and electro-mechanical systems"
SIAM J. Appl. Math. 35(1978) pp. 21-30 (with T. D. Morley).

In recent years the concept of parallel addition and other new operations have been introduced. These operations are derived from network interconnections involving Kirchoff's and Ohm's laws. From electrical considerations the domains of these operations, for the most part, have been restricted to positive semi-definite operators on a finite dimensional inner product space. In this paper we consider general electro-mechanical systems of the form: given a complex linear transformation $A : \mathcal{K} \to \mathcal{W}$ and a complex linear operator Z on $\mathcal{U}$, we seek a linear operator $\Psi(Z)$ on the range of A, such that for any $b \in$ range (A), there is a solution x, v to $Ax = B$, $Zx - A^*v = 0$ with $\Psi(Z) = v$. We term Z almost definite if $(Zx,x) = 0$ only when $Zx = 0$. If Z satisfies this condition, then $\Psi(Z)$, termed the transfer resistance, exists and is unique. Various properties are developed. In particular, an operator Z is almost definite if and only if $\Psi(Z)$ exists for all possible A. If Z is almost definite, then there is a complex constant α with $|\alpha| = 1$, such that $Re(\alpha Zx,x) \geq 0$, for all $x \in \mathcal{U}$. This model extends the domain of validity of the new operations and unites the theory.

36. "Inequalities induced by network connections" Recent Applications of Generalized Inverses, Editor, N. Z. Nashed, Pitway, London (1980), (with T. D. Morley).

It has been found that interesting mathematical relationships arise from a vectorial generalization of Kirchhoff's and Ohm's laws, in which the "resistors" become positive semi-definite (PSD) linear operators. In analogy to the parallel connection of resistors Anderson and Duffin stidied the parallel sum $P : S$ of two PSD operators on a finite dimensional space, defined by $R : S = R(R+S)^{+}S$. This paper extends the results of Anderson and Duffin that $\|R:S\| \leq \|R\|:\|S\|$ and $tr[R:S] \leq trR:trS$ to a wide class of operations derived by a vectorial analog of Kirchhoff's and Ohm's laws. These inequalities remain true in Hilbert space.

37. "Computational methods for solving static field and eddy current problems via Fredholm integral equations" IEEE Trans. on Magnetics 15(1979) pp. 1075-1084 (with J. H. McWhirter J. J. Oravec and P. J. Brehm).

Two-dimensional static field problems can be solved by a method based on Fredholm integral equations (equations of the second kind). This has numerical advantages over the more commonly used integral equation of the first kind. The method is applicable to both magnetostatic and electrostatic problems formulated in terms of either vector or scalar potentials. It has been extended to the solution of eddy current problems with sinusoidal driving functions. The application of the classical Fredholm equation has been extended to problems containing boundary conditions: (1) potential value, (2) normal derivative value, and (3) an interface condition, all in the same problem. The solutions to the Fredholm equations are single or double (dipole) layers of sources on the problem boundaries and interfaces. This method has been developed into computer codes which use piecewise quadratic approximations to the solutions to the integral equatiosn. Exact integrations are used to replace the integral equations by a matrix equation, The solution to this matrix equation can then be used to directly calculate the field anywhere.

38. "Inequalities induced by network connections II. Hybrid connections' J. of Math. Anal. and Appl. 67(1979) pp. 215 - (with T. D. Morley).

It has been found that interesting mathematical relationships arise from a vectorial generalization of Kirchhoff's and Ohm's laws, in which the "resistors" become Hermitian positive semidefinite (PSD) linear operators. In analogy to the parallel connection of resistors Anderson and Duffin studied the parallel sum $R : S$ of two PSD operators on a finite dimensional space, defined by $R : S = R(R+S)^{-1}S$. Duffin and Trapp then studied the hybrid connection. This paper generalizes some of their results to a much broader class of electrical connections.

39. "Wang algebra and matroids"
IEEE Trans. on Circuits and Systems, 9(1978) pp. 755-762 (with T. D. Morley).

Wang algebra is defined by three rules: i) $xy = yx$; ii) $x + x = 0$; and iii) $xx = 0$. K. T. Wang showed that these rules give a shortcut method for finding the joint resistance (or driving point resistance) of an electrical network. However, there are electrical systems more general than the Kirchhoff network. For these systems regular matroids replace networks. It is shown in this paper that Wang algebra is an excellent tool to develop properties of networks. Moreover the Wang shortcut method can still be used to find the joint resistance of an electrical network.

40. "An elementary treatment of Lagrange multpliers"
<u>Extremal Methods and Systems Analysis</u>, Ed. by A. V. Fiacco and K. O. Kortanek, Springer-Verlag, Berlin , 1980 pp. 357-373.

The problem of finding the infimum of a convex function $f(x)$ subject to the constraint that one or more convex functions $g(x)$ be non-positive can be treated by the Lagrange multiplier method. Such a treatment was revived by Kuhn and Tucker and further studied by many other scientists. These studies led to the following associated maximizing problem on the Lagrange function, $L = f(x) + \lambda\, g(x)$. First find the infinum of L with respect to x and then take the supremum with respect to λ, subject to $\lambda \geq 0$. The minimizing problem and the associated maximizing problem are termed dual programs. This paper is partly of an expository nature: The goal is to give a short and elementary proof that, under suitable qualifications, the infinum of the first program is equal to the supremum of the second program. The proof begins by using the Courant penalty function. No knowledge of linear programming is assumed. However, the duality theorem for linear programs is a special case of the duality theorem for convex programs developed.

41. "Clark's theorem on linear programs holds for convex programs" Proc. Natl. Acad. Sci. USA 75(1978) pp. 1624-1626.

Given a linear minimization program, then there is an associated linear maximization program termed the dual. F. E. Clark proved the following theorem. "If the set of feasible points of one program if bounded, then the set of feasible points of the other program is unbounded." A convex program is the minimization of a convex function subject to the constraint that a number of other convex functions be nonpositive. As is well known, a dual maximization problem can be defined in terms of the Lagrange function. The dual objective function is the infimum of the Lagrange function. The feasible Lagrange multipliers are those satisfying: (i) the multipliers are nonnegative and (ii) the dual objective function is not negative infinity. It is found that Clark's Theorem applies unchanged to dual convex programs. Moreover, the programs have equal values.

42. "A class of optimal design problems with linear inequality constraints" Instituto Lombardo (Rend. Sc.) A 112 (1980) (with B. D. Coleman, and G. P. Knowles).

The optimal design of a structure often requires the minimization of a linear functional subject to integral inequality constraints. In this note, cases are treated in which the constraints are linear. One such problem is the long vertical cables of variable cross-section. Another is the cantilevered beams of variable width. The relation to linear programming theory is developed.

43. "Operator networks treated by Sylvester unisignants" (with T. D. Morley)

Sylvester proposed a special format for writing a system of n linear equations on n variables. It resulted that the matrix elements of the inverse are unisignants. A unisignant is a rational function of the n^2 coefficients formed by addition, multiplication, and division, Subtraction is not permitted. We extend his theory to the case in which the coefficients are linear operators rather than scalars. The equations defining the steady flow of current through a network of conductors is a Sylvester system. By use of unisignants we generalize several properties of scalar networks to the operator case.

44. "Linear Inequality Systems Treated by Determinants" (with J. T. Buckwalter).

This study concerns a linear inequality system defined in terms of a matrix A. Fourier proposed to solve such a system by eliminating variables one at a time. Here Fourier's method is generalized so as to be able to eliminate variables in blocks rather than one by one. This solution is expressed in terms of certain minors of A. The dual problem of a system of equations with matrix A^T and positive variables is also studied. The solution is expressed in terms of the same minors of A by an algorithm analogous to Cramer's rule.

45. "Lagrangean Functions and Affine Minorants" (with R. G. Jeroslow).

We give hypotheses, valid in reflexive Banach spaces (such as L^p for $\infty > p > 1$ or Hilbert spaces), for a certain modification of the ordinary lagrangean to close the duality gap, in convex programs with (possibly) infinitely many constraint functions.

Our modification of the ordinary lagrangean is to perturb the criterion function by a linear term, and to take the limit of this perturbed lagrangean, as the norm of this term goes to zero.

We also review the recent literature on this topic of the "limiting lagrangean."

46. "Some problems arising from mathematical models" Constructive Approaches to Mathematical Models, Ed. by C. V. Coffman and G. J. Fix, Academic Press Inc. 1979 pp. 3-32.

In addition to previous models this paper describes the following problems:

(1) Transmission of signals without distortion.
(2) Wave motion and the Paley-Wiener theorem.
(3) Sylvester unisignant operators.
(4) A determinant algorithm for linear inequalities.
(5) Clark's theorem extended to convex programs.

47. "The fundamental mode of vibration of a clamped annular plate is not of one sign" Constructive Approaches to Mathematical Models, 1979 pp. 267-277. (with C. V. Coffman, and D. H. Shaffer).

We are concerned here with the modes of vibration of a clamped annular plate; that is to say, with the eigenvalue problem

$$\Delta^2\varphi = \lambda\varphi \quad \text{in} \quad D_\epsilon, \quad \varphi = \frac{\partial\varphi}{\partial n} = 0 \quad \text{on} \quad \partial D_\epsilon \ , \qquad (1.1)$$

where

$$D_\epsilon = \{z = x + iy : \epsilon < |z| < 1\}, \quad \epsilon > 0 \ .$$

and $\partial/\partial n$ denotes differentiation with respect to the outer normal.

Apparently is was A. Weinstein who first raised the question of whether the fundamental mode of vibration of a clamped plate must in general be of one sign. Interest in this question increased when Szegö showed that an affirmative answer would imply that among all plates of a given area the fundamental eigenvalue is smallest for a circular plate.

48. "Theory of monotonic transformations applied to optimal design problems"
Archive for Rational Mech. and Anal. 72(1980), 381-393 (with B. D. Coleman and G. Knowles).

The optimal design of a structure often requires the minimization of the value of an objective functional f over a set G of non-negative functions ζ which obey a constraint of the form $T\zeta(y) \leq \zeta(y)$, with T an integral operator. In a recent article we discussed cases in which the functional f and the operator T are affine functions. Here we present a method which can be useful when f and T are not affine but are monotonic in the sense that, for each pair of functions ζ_1 and ζ_2 in G obeying $\zeta_2(y) \geq \zeta_1(y)$ for almost all y, there holds $f(\zeta_2) \geq f(\zeta_1)$ and $T\zeta_2(y) \geq T\zeta_1(y)$. The method rests on a theorem of the following type: Under the assumptions that T is continuous in an appropriate sense and that the feasible set G, although not empty, does not contain the zero function 0, the infimum of f over G is attained at a function ψ, which is simultaneously a minimizer of f and a fixed point of T, can be obtained by successive applications of T to the zero function, i.e., $\psi = \lim T^n 0$, for almost all y, $T^n 0(y) \uparrow \psi(y)$. If the fixed point of T is unique, then, for each ζ in G, $\psi = \lim T^n\zeta$, and $T^n\zeta(y) \downarrow \psi(y)$ for almost all y.

49. "The limiting Lagrangean"
(with R. G. Jeroslow).

A somewhat modified form of the lagrangean closes the duality gap in convex optimization, in many circumstances when the ordinary lagrangean and the augmented lagrangeans leave a duality gap. For example, the duality gap for a consistent program is always zero using this modified lagrangean, when the objective function and constraints are closed, convex functions; in other instances, there are constraint qualifications, but these are typically weaker than the usual Slater point requirements.

50. "Are adobe walls optimal phase shift filters?" Advances in Applied Math (1980) (with C. V. Coffman and G. P. Knowles).

The adobe house construction gives an automatic, air conditioning effect because the rooms tend to be cool at midday and warm at night. Presumably this is brought about by the walls acting as a heat filter so that there is nearly a twelve hour phase lag. This raises the question of how to optimize the adobe phenomena by a suitable design of the walls.

In this study it is supposed possible to make the walls of layered construction with layers having different thermal resistivity. Such a layered wall can be modeled electrically as a ladder filter of capacitors and resistors. The input to the ladder is a sinusoidal voltage. Then the following question arises. If the filter capacitors have given values how should the resistors be chosen so that the output voltage has a given phase lag but least attenuation?

It is found possible to answer this question by use of a special variational principle. Applying this analysis to building construction shows how to maximize oscillation of interior temperature with a phase lag of a prescribed number of hours.

51. "A limiting Infisup theorem" (with C. E. Blair, and R. G. Jeroslow).

We show that duality gaps can be closed under broad hypotheses in minimax problems, provided certain changes are made in the maximin part which increase its value. The primary device is to add a linear perturbation to the saddle function, and send it to zero in the limit. Suprema replace maxima, and infima replace minima. In addition to the usual convexity-concavity assumptions on the saddle function and the sets, a form of semi-reflexivity is required for one of the two spaces of the saddle functions.

A sharpening of our result is possible when one of the spaces is finite-dimensional.

A variant of the proof of the previous results leads to a generalization of a result of Sion, from which the theorem of Kneser and Fan follows.

52. "Bounds for the r^{th} characteristic frequency of a beaded string or of an electrical filter"
Proc. Nat. Acad. Sci. (198[illegible]) (with M. F. Barnsley).

The fundamental mode of vibration of a beaded string has a shape without change of sign. The r^{th} higher normal mode of vibration has r changes of sign. Given any virtual shape of the string with r changes of sign, an algorithm is found which gives upper and lower bounds for the r^{th} characteristic frequency as a function of the virtual shape. By making a transformation it is found that this algorithm holds for the characteristic frequencies of an L-C network. Other transformations show that it applies to the r^{th} eigenvalue of a Hermitian matrix.

53. "Temperature control of buildings by wall design"
(with C. V. Coffman, and G. Knowles).

The adobe house developed in the hot arid climate of the American Southwest has the virtue of being cool in the day and warm in the night. The adobe wall acts as a filter giving nearly a twelve-hour phase lag in the outside temperature oscillation. However, on reaching the inside, the oscillation suffers a strong attenuation in amplitude. In a previous paper it was shown that if the resistivity of the wall could be varied in a certain way from outside to inside then the attenuation would be considerably reduced. In this paper a formula is given for the minimum possible attenuation under given design restrictions. Then it is shown how to approximate the optimum wall by constructing a wall with two or more layers of different materials. Numerical illustrations are given.

54. "On the structure of biharmonic functions satisfying the clamped plate conditions on a right angle"
(with C. V. Coffman).

Let $u(r,\theta)$ be biharmonic and bounded in the circular sector $|\theta| < \frac{\pi}{4}$, $0 < r < \rho$ $(\rho > 1)$ and vanish together with $\frac{\partial u}{\partial \theta}$ when $|\theta| = \frac{\pi}{4}$. We consider the transform $\hat{u}(p,\theta) = \int_0^1 r^{p-1}u(r,\theta)dr$. We show that for any fixed $\theta_o u(p,\theta_o)$ is meromorphic with no real poles and cannot be entire unless $u(r,\theta_o) \equiv 0$. It follows then from a theorem of Doetsch that $u(r,\theta_o)$ either vanishes identically or oscillates as $r \to \infty$.

55. "Puzzles, Games and Paradoxes"
Mathematics Department Booklet. Carnegie-Mellon University, January, 1979.

Questions for a problem seminar in applied mathematics. Most of the questions are variants of old problems but some are new.

56. "On the relationshop between the genus and the cardinality of the maximum matchings of a graph"
Discrete Mathematics 25(1979) pp. 149-156 (By T. Nishizeki).

Lower bounds on the cardinality of the maximum matchings of graphs are established in terms of a linear polynomial of $p, p^{(1)}, p^{(2)}$ and γ whose coefficients are functions of k, where p is the number of the vertices of a graph, $p^{(1)}$ the number of the vertices of degree $i (i = 1,2)$, γ the genus and k the connectivity.

57. "Lower bounds on the cardinality of the maximum matchings of planar graphs"
Discrete Mathematics 28(1979) (By T. Nishizeki and I. Baybars).

Lower bounds on the cardinality of the maximum matchings of planar graphs, with a constraint on the minimum degree, are established in terms of a linear polynomial of the number of vertices. The bounds depend upon the minimum degree and the connectivity of graphs. Some examples are given which show that all the lower bounds are best possible in the sense that neither the coefficients nor the constant terms can be improved.

58. "Linear Programming Supply Models for Multihour Traffic Networks"
Modeling and Simulation, edited by M. H. Mickle and W. G. Vogt, Socio-Eco. Systems, Instrument Society of America 10, part 4, 1431-1435, (By P. R. Gribik, K. O. Kortanek, D. N. Lee and G. C. Polak).

Marginal Investment costs are assumed known for circuits and switching facilities employed to supply telecommunications services. Posson generated customer demands are specified between junction pairs in a network hierarchy by times of day. A linear programming supply model is presented for computing circuit group sizes.

59. "Hewlitt-Nachbin spaces"
North-Holland/American Elseview, Amsterdam 1975 (By Maurice Weir).

The class of Hewitt-Nachbin spaces has won the interest of many point-set topologists in the past quarter-century. This book summarizes the recent progress. It serves to classify and to solve problems in a variety of mathematical discplines.

60. "Mathematical Methods in the Social and Managerial Sciences"
John Wiley and Sons, Inc. 1975 (By Patrick Hayes).

An introduction to differential equations, convexity, convex programming and geometric programming. Many of the problems are posed in terms of business or economic concepts.